Problem **20**

In this problem, we have a mass spring damper system with the respective mass, damping constant and stiffness. And we're forcing it with this mechanism over here, this mechanism essentially has an omega of four radians per second. So that's going to be our forcing frequency, we're asked to find the damping ratio, the phase angle of the city state solution, the natural period of oscillation, the period of the steady state response and the period of the damped vibration Taotie. So this is just a simple mass spring damper system. So we can use the equations that are provided in the formula sheet without having to derive them. But I'll discuss each of these equations in a bit more detail. So let's start with the damping ratio. So the damping ratio is defined as C divided by C C, which is equal to c divided by two times m times the natural frequency omega n. And the natural frequency omega n is equal to the square root of k over m, the stiffness divided by the mass. So we can actually solve for this natural frequency, this is going to be equal to the square root of 39 newtons per meter, divided by 5.2 kilograms. And this will be equal to 2.739 radians per second, completing this into the above equation, where c is equal to eight Newton's seconds per meter, divided by two times 5.2 kilograms times 2.739 radians per second. This is going to be equal to 0.2808. And this is our damping ratio. So this is the first part to our final answer. Now, let's look at the phase. So the phase phi is equal to arctangent of the following expression, two times the damping ratio, times the ratio of the forcing frequency over the natural frequency divided by one minus this ratio of the forcing frequency divided by the natural frequency, all squared. And if we plug this in, we get the following results, two, times 0.2808 times four, and forcing frequency that's four radians per second, given the question divided by 2.739 divided by one minus four divided by 2.739. All squared, we got that phi is equal to negative 0.627. radians. Really important that we remember it is in radians, not degrees. And we can convert this into degrees, but that is in radiants. Next up, we are looking at the period, there's different time periods, we're going to end analyze them all. So first, we need to look at the period of the natural period of oscillation tn, right. So this is if there wasn't any forcing, if we let it vibrate, what is the natural period and this has to do with the natural frequency. So the natural period tau n is equal to two Pi over the natural frequency. And we have the natural frequency that is equal to 2.739. So our natural period is 2.294 seconds. And this is again part over final answer. Next up, we're analyzing the time period of the steady state response tau naught. So tau naught deals with a steady state response. What is the steady state responsible? That's just the the forcing frequency right? Because after we start the system, we reached the forcing frequency so that's just to pay I divided by omega naught, right? And this is equal to two pi divided by four, which is equal to 1.571 seconds. And this is again part of our final answer. May last time period that we're going to analyze is tau dt. And this is the period of the damped vibration. What is this equal to this is going equal to two pi divided by the damped frequency. Now, we don't know the damn frequency, but we can derive it omega d is equal to the natural frequency times the square root of one minus the damping ratio squared. If we plug in the damping ratio in the natural frequency, we see that the damped and the natural frequency we see the dam frequency is 2.6 to nine radians per second, so two pi divided by 2.629 is equal to 2.390 seconds. And this is the final answer for the damped time period.